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RELATIONAL THINKING AND PROBLEM SOLUTION STRATEGIES BEGINNING ALGEBRA HIGH SCHOOL STUDENTS

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ABSTRACT
Many students' difficulties in learning early algebra are associated with deficiencies in learning during the student's transition from arithmetic to algebra. relational thinking; a way of thinking that can be a gateway to algebraic thinking, is touted as one of the solutions to these difficulties. This study aims to determine the profile of relational thinking and early algebra problem solving strategies for junior high school students. Through the phenomenological design, the researcher selected six participants who fit the research criteria. The data analyzed refers to several theoretical frameworks on relational thinking from several experts. This study focuses on the indicators of interpreting the equals sign , using the relationship between numbers and applying the basic properties of number operations, as well as the methods students use in solving initial algebraic problems (algebraic forms and linear equations of one variable). As a result and its implication, students' relational thinking is seen in several items, namely the first 3 indicator items: each item is seen in S1, S4, and S6; The second 2 indicator items are visible on the S4; and the third 3 indicator items are seen in S1. The majority of subjects do computing dominantly and or apply the nature of operations based on procedural knowledge. Similarly, in solving early algebra problems, it was found that the dominant tendency of students to consider algebraic expressions from a procedural perspective rather than a structural perspective (based on mathematical concepts or structures). From these results, it is deemed necessary to develop relational thinking since students study arithmetic in elementary school, including by familiarizing students with number sentences in various forms and encouraging understanding of mathematical structures (relationships between elements and understanding of the nature of number operations).

Keywords: Algebra, early algebra, relational thinking, problem solving strategies.

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INTRODUCTION

The importance of algebra in mathematics has been expressed by many researchers. Jupri & Drijvers (2014) state that algebra is very important for achievement in other mathematical domains such as analytic geometry, calculus, and statistics. Therefore, initial algebra learning, which includes students' first steps in this domain, is certainly an important phase in algebra learning (van Amerom, 2002). Research on algebra is often done through the development of algebraic thinking, where this thinking is needed to analyze deeper mathematical structures (Kiziltoprak & Kose, 2017). Similarly, research on early algebra is often done through the development of early algebraic thinking.

The education curriculum in Indonesia has accommodated early algebra learning at the junior high school (SMP) level. Based on 2017 revised 2013 Curriculum, the basic material for initial algebra is found at the level of 7th-grade junior high school students in semester 1 which includes algebraic forms and linear equations and inequalities of one variable. The subject matter of algebraic forms includes: coefficients, variables, constants, and terms in algebraic form ; arithmetic operations in algebraic form; and simplification of algebraic forms. The subject matter of linear equations and inequalities of one variable includes: statements; open sentence; and solving one-variable linear equations and one-variable linear inequalities.

In Indonesia, students often experience difficulties in understanding early algebra, including difficulties related to the application of arithmetic operations (Jupri & Drijvers, 2014), students' difficulties in completing algebraic expressions (Muchoko et al., 2019), and difficulties in solving algebraic expressions. equation (Jupri, Drijvers, & Heuvel-panhuizen, 2014). Difficulties related to the application of arithmetic operations, for example students' errors in using the nature of arithmetic operations and interpreting the equal sign (Jupri & Drijvers, 2014).

Judging from the characteristics, there is a difference between arithmetic and algebra. Arithmetic is very close to calculation, while algebra focuses on relations (Carpenter et al., 2005). For example, in arithmetic, the equals sign often refers to calculating and writing numerical answers. For example, when faced with ..., students tend to get a specific answer 5 and do not view it as or (Jupri & Drijvers, 2014), whereas in algebra, the equal sign indicates a relation or has the meaning "algebraically equivalent to" (Herscovics & Linchevski, 1994). For example, when writing becomes (Jupri & Drijvers, 2014).

On the other hand, arithmetic and early algebra are closely related, especially in the nature of number operations. However, often students do not understand how the basic nature of number operations is applied in calculations, so as a result, students do not realize that arithmetic and algebra are based on the same basic ideas (Carpenter et al., 2005). For example, in arithmetic, number sentences will be very easy to complete if students subtract 56 from 56 first instead of doing calculations sequentially from left to right.

For some students, learning algebra is a natural progression that builds on their understanding of arithmetic (Harbour, Karp, & Lingo, 2017), however, for many students, learning algebra is completely different from the experience of learning arithmetic, and they find themselves experiencing a number of difficulties. in the transition (from arithmetic to algebra) (Cai & Moyer, 2008). The shortcomings resulting from the way of learning arithmetic during the transition from arithmetic to algebra have an influence on the development of algebraic thinking (Kiziltoprak & Kose, 2017). Therefore, the transition period of students from (thinking) arithmetic to (thinking) algebra becomes a vital phase in determining student success in mastering algebra well.

Relational thinking is said to be a bridge between arithmetic thinking and algebraic thinking (Kiziltoprak & Kose, 2017).

Relational thinking involves the meaning of the equals sign , the use of the basic properties of operations and number relationships (Carpenter et al., 2005), and making strategic decisions (Harbour et al., 2017; NCTM, 2017). Strategies that can be used include "takeaway" or taking 3 out of 100 or taking 41 out of 39. Another strategy, by viewing subtraction as a distance, is by counting backwards, for example counting from 3 to 100 or from 39 to 41. Students who choose the strategy " take away" on subtraction and choose a "distance" strategy to be believed to demonstrate relational thinking (NCTM, 2017).

(Kindrat & Osana, 2018) states that the center of algebraic reasoning is relational thinking, which involves coordinating quantities in mathematical expressions, often without calculation, using flexible reasoning about quantities and converting mathematical expressions into equivalents. For example, sentence numbers (Kindrat & Osana, 2018). The equal sign allows students to ignore the 1986 number on each side and only focuses on equations that allow students to reduce the computational level (Kindrat & Osana, 2018).

The hallmark of relational thinking is not determined by computation, but rather by mathematical structure and generalization (Carpenter et al., 2005). To illustrate, when faced with a problem such as 25 + 17 = 22 + , students who understand the equals sign meaning "equal to" can solve it either computationally (i.e., 25 + 17-22) or by relational reasoning, which would require checks the relationship between the sums on both sides of the equation (ie, "25 is 3 more than 22, so the answer must be 20 because 3 is greater than 17 to balance it."). In this context, relational thinking involves examining the equation, paying attention to the structural relationship between 25 on the left side of the equation and 22 on the right side, and compensating for differences by adjusting the number 17 (Kindrat & Osana, 2018).

Previous research has shown that students experience serious misconceptions about the meaning of the equal sign ("=") (Baiduri, 2015) Jupri & Drijvers, 2014). Students perceive the equals sign as an operator to perform calculations, state results, or as a signal to write down the next answer (Harbour et al., 2017; Jones & Pratt, 2012 ; Kiziltoprak & Kose; 2017). Other research on relational thinking related to the use of number sentences, either true or false or open number sentences, has brought much success in developing relational thinking (Banerjee, 2011) (Carpenter et al., 2005). A number of studies have also succeeded in designing relational thinking-based teaching, including through pre-service training for teachers (Fisher et al., 2019), as well as several projects related to the development of the teaching profession in terms of This has also shown positive implications for the development of students' relational thinking (M. L. Blanton & Kaput, 2005).

A number of researchers have also categorized relational thinking based on research findings to see students' level of relational thinking (Kiziltoprak & Kose, 2017; M. Stephens & Wang, 2008). One of them is Kiziltoprak & Kose (2017) which groups students based on the operation process carried out into three themes, namely the operation process based on relational thinking, consisting of sub-themes using the basic properties of arithmetic operations and using relations between numbers; the process of introducing operations to relational thinking, consisting of a sub-theme of explaining relations after finding the unknown and prerelational thinking; and results-oriented operating processes.

In solving a math problem, the curriculum encourages students to apply a variety of strategies, but more than just applying strategies, relational thinking involves making strategic decisions (NCTM, 2017), students integrate relational thinking strategies with computational skills so as to prepare students to succeed in advanced mathematics. (Harbour et al, 2017). In relation to problem solving strategies, (M. Blanton et al., 2019) through their strategic approach, divides relational thinking into three categories: structural strategies (compensation), computational strategies, and operational strategies. Therefore, relational thinking is closely related to problem solving strategies. mathematics and on this basis this research analyzes mathematical problem solving strategies and selects a preliminary algebraic topic.

(Baiduri, 2015) has researched relational thinking which focuses on understanding the equal sign and analyzing students' strategies in solving equivalence equations through written test analysis. This study analyzes relational thinking with a wider scope, namely the aspect of using the relationship between numbers, and applying the basic properties of number operations. This study also analyzes students' strategies in solving initial algebraic problems that are more varied (not only solving equivalent equations), which include several indicators with the topic of algebraic forms and one-variable linear equations. This research focuses on the analysis of relational thinking which includes three indicators (interpreting the equal sign as equality, using the relationship between numbers, and applying the basic properties of number operations) and analysis of strategies in the form of ways that students use when solving problems related to shapes. algebra and linear equations of one variable.

METHOD

This study aims to analyze students' relational thinking and early algebra problem solving strategies so that with qualitative methods it is hoped that complete and in-depth analysis results can be obtained. Judging from the characteristics of qualitative research according to Creswell (2017), this study uses natural conditions, namely students as direct data sources, and researchers are key instruments. The data collected tends to be in the form of words, namely from the results of written tests and student interviews about the number sentenced and initial algebra, in other words descriptive. This research also places more emphasis on the students' process of solving any given problem rather than the final result and tends to analyze the data inductively.

Data collection was carried out in two stages through two different methods. The first stage, to obtain initial data about students' relational thinking, was carried out a written test technique. This written test is given to students in the form of number sentence questions and initial algebra questions . So that researchers can see the emergence of indicators, in each number sentence a column is given to write down the reasons (justification) for the answers given. The second stage, to validate the results of the written test, conducted an interview technique. The interview technique is conducted online (in the network), namely by telephone. This is because when the interview has entered the COVID-19 pandemic period and the related school applies distance learning (online) so it is not possible for researchers to conduct interviews directly (face to face) at school . So that researchers can see the emergence of indicators, in each number sentence a column is given to write down the reasons (justification) for the answers given.

RESULTS AND DISCUSSION

A. Student's relational thinking in completing number sentences

Based on the results of the written test and relational thinking interview for each student/subject, the following are the results of the research found:

1. Subject S1

In interpreting the equals sign , S1 has understood it as equality or balance. Problem 38+45 = ... + 47 is answered correctly by first adding the number on the left side of the equals sign (38 and 45), then subtracting it from the number on the right side (47) as shown in Figure 4.1. This shows that the subject thinks that the sum of the numbers on the left side must be the same as the sum of the numbers on the right.

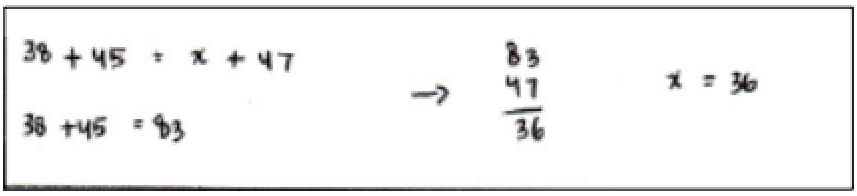

Figure 1. Answer 2.1 by S1

As for the second indicator, using the relationship between numbers or compensation, it has not been seen in the S1 subject's answer. All questions related to this indicator are solved by means of computation or ordinary calculations. For example, true or false number sentence 45+28 = 47+26

benar	karena hasil dari 45 + 28 = 47 + 26 (sama dengan) 73
salah	karena hasil 75 - 39 tidak sama dengan hasil 73 - 36.

Figure 2. Answers 1.5 and 1.6 by S1

From the answers above, subject S1 compares the results of operations on each side of the equal sign to determine whether the results are the same or not. That is, for these questions, the S1 subject has only just arrived at the understanding of the meaning of the equal sign as equality, but has not shown relational thinking in the second indicator.

For the third indicator, using the basic nature of number operations, subject 1 already has knowledge of the basic properties of number operations (commutative, associative, and distributive). Subject 1 understands the commutative nature as the nature of the exchange of numbers (based on his learning experience in elementary school) and uses it in solving questions about commutative indicators (questions 1 to problem 3). The subject believes that the truth value of these statements can be determined without having to do calculations. That is, subject 1 only performs calculations as a step to verify answers, not as the main step in completing number sentences 1.1-1.3 , as shown in Figure 3.

1	benar	karena soal ini memiliki sifat komutatif (penjumlahan) (pertukaran).	24 + 49 = 73 49 + 24 = 73
2	salah	di soal no. 2 tidak bersifat komutatif, karena komutatif tidak berlaku untuk pengurangan.	
3	benar	Sifat komutatif juga berlaku pada perkalian.	

Figure 3 Answers 1.1-1.3 by S1

The written answer above shows that subject 1 understands that the commutative property applies to the operations of addition and multiplication of numbers but does not apply to operations of subtraction of numbers.

Likewise for the distributive nature, the subject did not complete the number sentence 2.5 to completion (see Figure 4.4). Answer 2.5 shows that subject 1 assumes two unknown numbers as , which means that the subject thinks that the two numbers must be the same. Even though the dots given mean answers that might be different and it seems that S1 has not thought of this. This can be seen after being given instructions in the form of a general formula for the distributive property ( ) , the subject can solve it by giving one of the correct answers, namely 7 and 7.

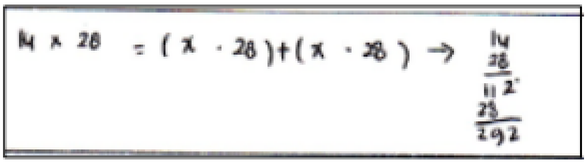

Figure 4. Answer 2.5 by S1

Thus, the subject of S1 has shown relational thinking in several indicators, namely interpreting the equal sign and applying the nature of number operations. The subject is able to interpret the equals sign as a sign of equality even though he has not been able to express it through words.

2. Subject S2

For the first indicator, interpreting the equal sign as equality, the written answer from S2 has shown this understanding as equality or balance, namely the equality of the results of number operations on each side of the equals sign. However, when interviewed, S2 only mentioned the meaning of the equal sign as “result” and had difficulty finding other meanings in words. In addition, there is a unique answer written by S2, he adds the numbers on the left side and gets the result 83, then writes 47 + ? = 83. This method is referred to by subject 2 as a “logic way”, not in a direct way 83 – 47. The final result or answer given by the subject is correct, but there are errors in writing, 47 + ? = 83 = 36, that 83 should not be equal to 36. The equal sign in this sentence shows the meaning of the equal sign as a symbol to write the final answer or a place to write the answer. In this finding, the final answer is still correct (36), it's just that there is an incorrect use of the equal sign in the completion process (see Figure 5).

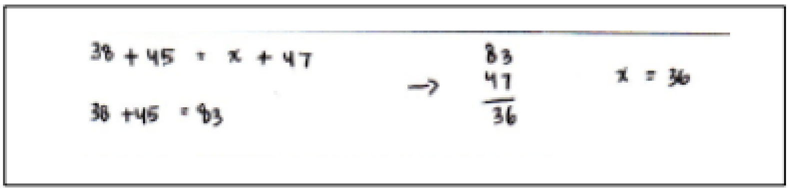

Figure 5 Answer 2.1 by S2

Indicators using the relationship between numbers or compensation have not been seen in subject 2. Like subject 1, S2 tends to do calculations for all number sentences related to this indicator. For example in problem 2.3 (see Figure 6). From this answer, implicitly the subject has applied commutative and associative properties in solving problems. However, they have not been able to find a simpler way , namely by looking at the relationship between the numbers 856 and 857. The incorrect use of the equal sign is also found in this answer.

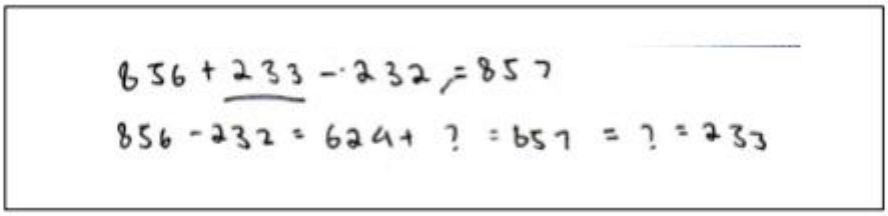

Figure 6 Answer 2.3 by S2

In applying the basic nature of number operations, subject 2 has been able to apply commutative and associative properties in solving problems, but the subject does not mention these properties as "commutative" and "associative" properties because they forget the terms. S2 correctly answered question 1.1 with a logical reason, "because it has the same number and sign". From the results of the interview, it means that the numbers are the same, as are the signs (positive/negative). S2 also added, "If the numbers are the same but the sign is different, the result will be different". Likewise for problem 1.2, the subject does not do the calculation because he believes that the result of the first operation of each side will be different, namely between 35-8 and 8-35. S2 includes a logical reason in the written answer, because if the smaller number is subtracted from the larger number, it will result in a negative. This implicitly states that the commutative property does not apply to addition.

The uniqueness of S2 is seen in solving problems related to associative properties. Problem 1.4 in the form of a true or false number sentence can only be solved by S2 by performing calculations according to the order of operations and admitting that they do not know any other way. However, in question 2.4 which is an open number sentence, the subject solves it by applying the associative nature (see Figure 7) even without a good knowledge base, in the sense that the subject does not know that the method he does is based on the associative nature.

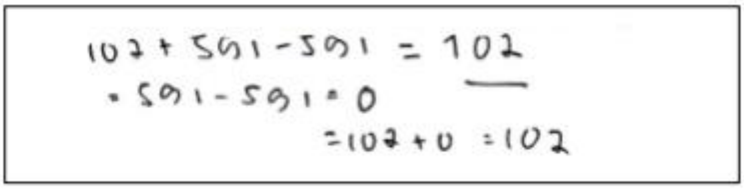

Figure 7 Answer 2.4 by S2

Based on the results of the analysis of each relational thinking indicator, the master's subject has understood the meaning of the equal sign as equality, but on the other hand, there are still many writings that indicate the equal sign as a symbol to write the results or answers that come next.

3. S3 Subject

From the snippet, S3 interprets the equal sign as "result", meaning a sign that shows the result. In addition, S3 also interprets it as a "sign of equality", meaning that the value or result of the operation of both sides of the equals sign is the same, as in the problem 24+46 = 46+24. That is, S3 calls 24+46 = 46+24 as an equation (should be equality). Thus, the researcher understands the meaning of this subject 3 that the equal sign indicates the result or indicates an equality.

In general, S3 subjects solve number sentence problems by calculating . Likewise, the arguments or reasons for the answers given are dominant in the calculation reasons, such as the equality of the results of the operation; and some logic, such as in the number sentence 24 + 49 = 49 + 24 S3 provides the argument "because if the addition is reversed the same , the number is still the same" (meaning that if the placement of the numbers is reversed, the number remains the same). Subjects also did not appear to use number linkages or compensation in solving questions related to this indicator.

Regarding the application of the basic nature of number operations, from the results of interviews with question 1.1, the subject believed that the value of the statement 24 + 49 = 49 + 24 was correct because it was only reversed by the placement of the numbers, as well as for multiplication. However, the subject is still unfamiliar with the term "commutative". Likewise with associative properties, S3 can only solve problems 102 + 591 – 591 = ... with sequential calculations (Figure 4.8). The subject said he did not know the basic properties of number operations. The subject also doubted whether in elementary school had learned about this topic.

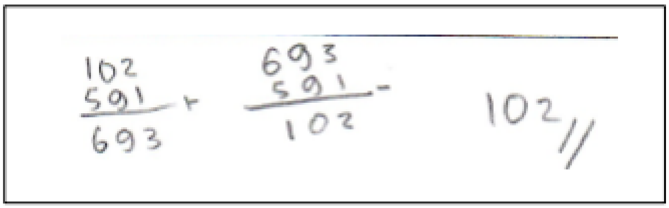

Figure 8 Answer 2.4 by S3

An interesting finding from the S3 subject is that when given an interrogative approach, in solving questions about indicators of distributive nature, the subject is able to find alternative answers (besides 7 and 7) for question 2.5, namely 6 and 8. The subject argues because the number is 14.

4. Subject S4

When the subject S4 was asked by the researcher what the meaning of the equals sign meant, he answered "comparison" or "result". The purpose of comparison according to subject 4, namely when compared the results between the right and left sides of the equal sign must be the same. From this answer, even though the word choice is not quite right, S4 has shown a fairly good understanding of the meaning of the equal sign as equality.

Further analysis related to answer 1.5 by S4 will be discussed in the discussion section (analysis per indicator). Likewise for problem 2.3, the subject can see the relationship between the numbers 856 and 857. However, S4 has not been able to apply compensation for questions with subtraction operations. Subjects were only able to perform ordinary calculations for the other two questions.

One of the problems regarding the application of associative properties is solved by S4 in two ways , namely by applying associative properties and sequential calculations (Figure 9). For the application of distributive properties, subject S4 is able to find the right solution by calculating and confirming the correctness of the answer by replacing (substituting) the empty part with the solution obtained.

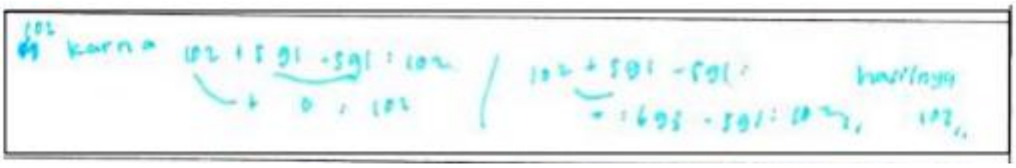

Figure 9 Answer 2.4 by S4

Thus, S4 has shown the ability of relational thinking in interpreting the equal sign as equality and is also able to use the relationship between numbers or compensation (for addition), but the subject has not shown a good understanding of the properties of number operations even though he has been able to apply commutative and associative properties.
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5. Subject 5

Seeing this written answer from S5, the researcher gets the first impression as a result of work that tends to be "original" (see Figure 4.10 left). However, after being dug through interviews, several unique findings were obtained from this S5. The subject is able to mention some basic properties of number operations such as commutative and associative in his argument, but has not shown the level of understanding. For example, for a number sentence with commutative indicators, the subject is able to mention the commutative reason which means exchange. However, when asked to explain further what he meant , the subject failed to provide an explanation and seemed still confused. That is, S5 has knowledge of the basic nature of number operations but understands them.

In the open number sentence 38+45 = ... + 47, the written answer is wrong, which is 38. The argument presented by S5 against this answer is incorrect and tends to be made up .. When explored through interviews, the reason is "using the distribution formula, associative". But again, the S5 has not succeeded in explaining to the researchers what it means. On the other hand, in interpreting the equals sign , S5 already understands and interprets it as equality. However, the second indicator of relational thinking, compensation, has not been seen in S5.

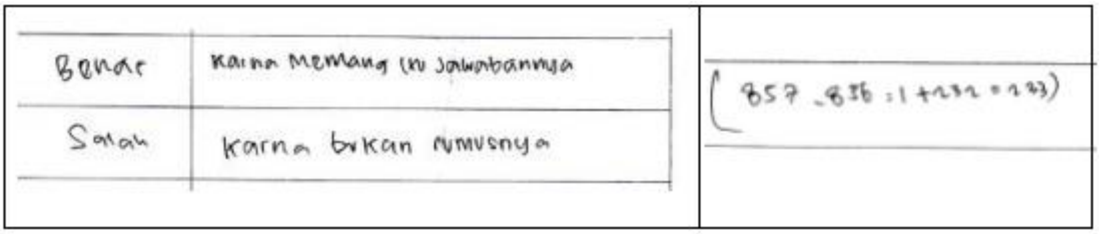


Figure 10 Answers 1.1, 1.2, and 2.3 by S5

Some of the argument columns in the number sentence looked empty, when confirmed, S5 had many questions that he forgot how to get the answer or solution he wrote. The unique findings of S5 are found in the solution of question 2.5, with indicators of distributive properties. S5 got the right answer, namely 10 and 4. When asked the reason, it was because 10 was the tens and 4 was the unit. Maybe this answer is influenced by previous learning experiences related to place value on numbers in elementary school.

6. Subject 6

From the results of the interview with S6, it was obtained information that S6 interpreted the equal sign as an equality of results or balance. For number sentences related to indicators using interrelationships between numbers or compensation, S6 is able to solve all questions with correct answers by means of ordinary calculations or calculations. The subject cannot resolve by means or other possible arguments to avoid computation. For example, for question 1.5, the following are the results of an interview with S6.

Likewise, for number sentences related to indicators using the basic nature of number operations, all problems are solved by dominantly result- equivalent-oriented calculations (Figure 4.11) and some logic. Thus, in general, S6 has shown a good ability to interpret the equals sign , but other indicators have not been seen. The subject of S6 is still very dominant in doing calculations in solving problems.

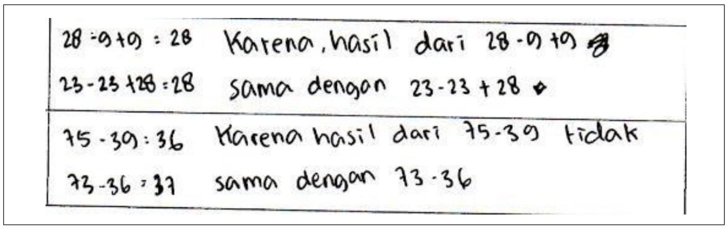


Figure 11 Answers 1.5-1.6 by S6

From the results of data analysis on the results of written tests and interviews of all subjects that have been described, in general the relational thinking of each subject as seen from each indicator can be summarized as shown in Table 4.1. The ST code indicates that the relational thinking indicator has been shown, the BT code indicates that the relational thinking indicator has not yet emerged, while the MT code indicates an introduction to relational thinking, namely subjects who (i) are able to interpret the equal sign as equality but there is still a tendency to use the equal sign as a symbol to write the next answer, (ii) able to use number relations or apply the basic nature of number operations but not based on proper knowledge or subject capable of avoiding or simplifying computations by using logical reasoning.

Table 4.1 Relational Thinking Subject per Indicator

INDICATOR	NUMBER QUESTION	*RELATIONAL THINKING*					
		S1	**S2**	**S3**	**S4**	**S5**	**S6**
1. Meaning sign "=" as equality	2.1	ST	MT	MT	ST	MT	ST
2. Using linkage between numbers or compensation	1.5	BT	BT	BT	ST	BT	BT
	1.6	BT	BT	BT	BT	BT	BT
	2.2	BT	BT	BT	BT	BT	BT
	2.3	BT	BT	BT	ST	BT	BT
	1.1 commutative summation	ST	MT	MT	BT	BT	MT
	1.2 no commutative subtraction	ST	MT	BT	BT	BT	MT

3. Using nature operation number	1.3 commutative and associative multiplication	ST	MT	BT	BT	BT	BT
	1.4 associative summation	BT	MT	BT	BT	BT	BT
	2.4 associative summation	BT	MT	BT	MT	BT	BT
	2.5 distributive	BT	MT	MT	BT	BT	BT
	2.6 distributive	BT	BT	BT	BT	BT	BT

B. Students' strategies in solving initial algebra problems

The following will describe the analysis of student responses in solving initial algebra problems, both from the results of written tests and interview results.

1. Subject S1

In writing, S1 was able to solve the first four questions of the six algebra questions given. The first problem with indicators using the properties of operations (commutative, associative, and distributive) in algebraic form has not been able to be solved correctly on all related problems. Problem 1.1 is answered correctly and accompanied by the right argument, namely that 2 = 2 is true because the commutative property of multiplication applies. As for the completion of sections 1.2 and 1.3, several errors were found in understanding algebraic expressions, such as + 5 = 5 , (3 +) = 3 . It can be concluded that the strategy used by S1 in the first question is to apply the properties of operations to algebraic form. Unfortunately, S1 does not yet have a good understanding of some algebraic expressions as shown in Figure 4.12.

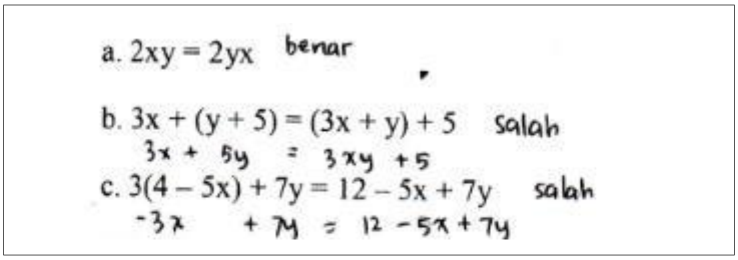

a. $2xy = 2yx$ benar
b. $3x + (y + 5) = (3x + y) + 5$ salah
$3x + 5y = 3xy + 5$
c. $3(4 - 5x) + 7y = 12 - 5x + 7y$ salah
$-3x + 7y = 12 - 5x + 7y$

Figure 12 Answers to the Algebra Test no.1 by S1

The last two questions about solving problems related to linear equations of one variable were not answered in writing by S1.

2. Subject S2

In solving the first problem about algebraic forms, S2 tends to focus on the given operation. In the first part, the subject did not understand the meaning of " " and " " . The second part was said by S2, "The method is the same, both are multiplied, the result will be the same". The subject thought that brackets in algebraic form always meant multiplication. In this problem, the brackets only show the grouping. Likewise for the third part, according to S2 because the left side has multiplication, the right side is addition, the subject concludes "the method is different , the results are different". operations that apply to him.

1. a: benar , karena dua-duanya memiliki huruf yang sama
b: benar, karena memiliki cara yang sama
c: salah, karena berbeda caranya

Picture 13 Answer Algebra Test number 1 by Subject 2

As for the second question, from the written answer the subject answered "different" but not accompanied by a clear reason. When confirmed through the interview, the subject sounded confused, he tried to rework so that he got 3 + 7 3 + 3 = 15 3 + 3 , then 3 + 7 = 15. At this point, the researcher tried to give direction whether this equation is the same with equation (i) and he answered the same. Finally , the subject concludes that in both equations has the same value . Thus , S2 quite understands the concept of balance in algebraic equations, but procedurally it seems that he has not mastered how to solve equations well. This is supported by written data for questions about finding variable values. The correct answer from S2 is only one of three parts and this correct answer is obtained from an inaccurate process (Figure 14).

4. a. x - 3-3 = 15 - 3
= 18

Figure 14 Answers Algebra 4.a by Subject 2

Figure 4.14 shows that the final answer written by S2 is correct, but there is a wrong process, namely when S2 tries to simplify the algebraic form by subtracting 3 on both sides. This is supported by the next answer to the problem of finding the value of from equation 2 3 = 15 as shown in Figure 4.15. The subject tries to find a solution to the equation by trying to replace the variable with a number (substitution).

Figure 15 Algebra 4b by Subject 2

5. 2 + 3 = 5
15 - 6? = 5
? = 10

The last two problems were not answered by S2 on the grounds that there was not enough time. However, from the results of the interview, the subject was able to understand the meaning of questions 5 and 6 and was able to make an algebraic model for the length of the rectangle at number 5 with a little direction from the researcher. For the last question, the subject can also give verbal and relatively spontaneous answers by using the "guess and check" strategy.

DISCUSSION

1. Student's relational thinking in completing number sentences

Based on the findings that have been described in the previous section, the relational thinking analysis of the subject seen from each indicator is as follows.

a. Define the equals sign (=)

When dug deeper into each subject by referring to the results of the written test (especially question 2.1), all these words tend to refer to the meaning of the equal sign as equality or balance. There is no subject who writes the answer 83 in the number sentence 38+45 = ... +47 as the meaning of the equal sign as arithmetic-specific and non-relational as stated by (Pang & Kim, 2018). The word "result" uttered by the subject turned out to be not completely meaningful as stated by (Warren, 2006), namely as a sign to find results or as a symbol before writing an answer. Because the subject also understands the equal sign by showing the same result between the results of the operation to the right of the equal sign and the result of the operation to the left of the equal sign. This is also supported by the correct answer in the sentence number 2.1. On the other hand, there are findings in the written answers of several subjects that indicate the meaning of the equal sign as a symbol used before writing the answer or writing the next answer, for example answers 2.1-2.3 by S2 as shown in Figure 4.21.

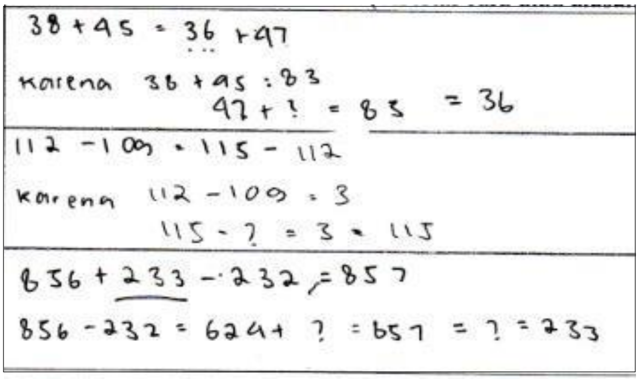


Figure 21 Answers 2.1-2.3 by S2

The answer above shows that the subject of S2 has implicitly understood the meaning of the equal sign as equality, but there is still a tendency to use the equal sign as a symbol to write the next answer. Another example is answer 1.2 of S3 (Figure 4.22).

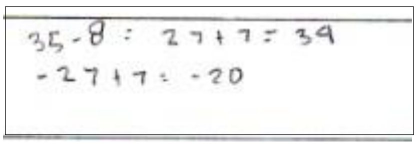


Figure 4.22 Answer 1.2 by S3

In general, the appearance of this indicator tends to be uniform in all subjects. This finding may be partly due to the maturity factor of the subject who is already in grade 7 of junior high school, while the majority of similar studies on this indicator were conducted on elementary school age , grade four to grade 6. For example, research by Falkner et al. (1999); Kieran (1981); and (Molina & Ambrose, 2006) who showed that the majority of 6th grade elementary school students had difficulty finding answers to non-canonical expressions such as 8+4=...+5.

b. Using the relationship between numbers or compensation

In general, the research data indicate that almost all subjects have not been able to use the relationship between numbers or make compensation in completing number sentences

From the written data, all subjects used the concept of the similarity of results or the concept of the balance of the equal sign . However, from the interview data, it was found that in addition to the usual computational methods on the written test, S4 subjects were able to use the relationship between numbers or compensate as another way of solving problems 1.5 and 2.3. For example, for a number sentence 1.5, 45+28=47+26, the subject explains, "The first one is 45 plus 28, the second is 47 plus 26. The 28 is, the 2 are 45-in, so 47 adds 26 , that's the same." However, the researcher tried to visualize the statement of subject 4 as follows:

$$\overset{2}{\curvearrowleft}$$

$$45 + 28 = 47 + 26$$

$$(45{+}2) + (28{-}2) = 47 + 26$$

$$47 + 26 = 47 + 26$$

This way of thinking of subject 4 implicitly uses the relationship between the numbers 28 and 26, i.e. 28 is 2 more than 26, which is then used to change the similarity of numbers so that the same form is obtained with the number to the right of the equals sign. According to (Carpenter et al., 2005), a student who considers an expression or equation as a whole may notice that one number is more or less than another number.

Some subjects believe that there is no other way to solve the problem, this is in line with the findings of Kiziltoprak & Kose (2017), namely that there are students who think they will not be able to find the numbers to be written in the related boxes (dots) without doing calculations . The reason for this situation is considered to be the fact that students learn arithmetic on a result-oriented basis and that they focus on calculations rather than on the relationship between numbers and operations (Kiziltoprak & Kose, 2017).

c. Apply nature base number operation

Students' learning experiences also seem to have a great influence on this indicator, such as the subject of S1 whose appearance of the relational thinking indicator is most visible in this indicator, he has existing knowledge about the basic properties of number operations which he obtained in elementary school.

In general, the findings of the written test and interview data on these third indicators can be categorized into four categories. First, relational thinking, which is the subject of applying the basic nature of number operations with sufficient knowledge about the basic nature of number operations. Second, the subject applies the nature of number operations without sufficient knowledge or the subject is able to avoid doing calculations based on logical reasoning. Third, the subject knows the nature of number operations but has not been able to apply it in solving problems. Fourth, the subject does not know so that he is unable to apply it in solving the problem. This categorization briefly can be seen in table 4.3.

Table 4.3
Categorization of Indicators Applying Basic Properties of Number Operations

Knowledge - Application	**Know**	**Not Know**
Capable apply	I	II
Not able to apply	III	IV

In relation to the data in Table 4.1, category I (knows and is able to apply) is included in relational thinking (ST), category II (able to apply, but does not know) and category III (knows but cannot apply) is included in the introduction to relational thinking (MT), while category IV (don't know and don't apply) is included in the unseen relational thinking. The basis for this grouping is the various theoretical frameworks that discuss relational thinking that leads to the selection of strategies and how these strategies are executed (Baiduri, 2015) (Harbour et al., 2016), but with the right foundation of conceptual knowledge. rather than just descriptive knowledge or procedural thinking ("Equality Relation and Structural Properties," 2010). This is because the goal of relational thinking is an understanding of why a method or strategy may or may not be implemented or applied and this requires an understanding of the nature of number operations (Carpenter et al., 2005). Therefore, categories II and III cannot be categorized into relational thinking, but because they can be a good foundation for the development of relational thinking, in this study, they are classified as an introduction to relational thinking.

From Table 4.1, for the three indicators, it can be seen from 42 items that only 3 items are indicated as relational thinking (ST), 11 items are included in the category of introduction to relational thinking (MT), and the remaining 28 items have not shown relational thinking. The three items of relational thinking that appear are limited to the application of commutative and associative properties, while for number sentences with the application of distributive properties, all subjects tend to do calculations. Open number sentences $14 \times 28 = . . . \times 28 + . . . \times 28$ in this study tends to produce only answers 7 and 7 and students generally cannot recognize the form of the question as $(+) \times = (\times) + (\times)$ so they cannot find other answers. This is in line with Kiziltoprak's research (2017) which found that the biggest difficulty of students was in applying the distributive nature. have also conducted research on relational thinking that focuses on the distributive nature.

This has been revealed by Kiziltoprak & Kose (2017), namely students are not familiar with mathematical expressions such as number sentences that involve operations on both sides of the equation due to the limited number of student books that provide such number sentences. In fact, completing number sentences can improve relational thinking (Carpenter et al., 2005). Furthermore, (Banerjee, 2011) concluded that arithmetic with number sentences is very useful for bringing students to algebra.

Based on previous studies, the dominant factor influencing the development of students' relational thinking is the teacher's learning design (Carpenter et al., 2005). (Carpenter et al., 2005) used the term scaffolding to refer to the way teachers develop students' relational thinking framework, while Kiziltoprak & Kose (2017) mention the term teaching interrogative approach to refer to questions that lead students to develop their relational thinking. Thus, due to the importance of teacher-made teaching designs, many studies recommend training for teachers to implement relational thinking-based teaching. This can be in the form of web-based professional development programs and in-service training (Kiziltoprak & Kose, 2017), or pre-service training for teachers to develop relational thinking-based teaching (Fisher et al., 2019).

Several projects related to the development of the teaching profession in this regard have also been carried out (M. L. Blanton & Kaput, 2005). Students taught by participating teachers showed significantly better understanding of the equals sign and used relational thinking during interviews than students taught by non-participating teachers (Jacobs et al., 2007).

2. Strategy Solution Problem Algebra Beginning
   a. Initial algebra problems related to the application of the properties of operations in algebraic form (questions 1 and 3)

In determining the truth value of $2 = 2$, all subjects were able to determine that the expression was true by just looking at its shape, but only a few of them were based on the right reasons. Among them there are also those who still misunderstand simple algebraic forms such as saying that 2 times is denoted by squared (S5 and S6). Only S1 shows a good understanding of the application of this commutative property. For the application of the associative property, $3 + (+ 5)$ = $(3 +) + 5$, S2 and

S3 answered correctly that the statement was true, while the other subjects stated that the statement was false. For example, S2 states that "this statement is true because the method is the same , both are multiplied". That is, S2 understands that brackets "()" in algebraic form always mean multiplication and he ignores the existing addition sign so that he concludes so. It seems that the associative nature of this algebraic operation is still "foreign" for all subjects.

As for the application of the distributive property in algebraic operations, $3(4 – 5) + 7 = 3.4 – 5 + 7$, the majority of subjects have actually known this property by calling it "rainbow times", but not all subjects recognize that this statement involves the application of those properties. Only S3 answered correctly accompanied by the right process even though it still included procedural knowledge, in the sense that S3 did not really understand this as part of the properties of operations on algebraic forms, but only carried out a series of procedures based on previous learning experiences.

Uniquely, although they both involve the application of operational properties in algebraic form, many errors are found in questions in the form of true and false statements, while for algebra questions in the form of determining variable values from equations, some subjects tend to be able to solve equations 4a and 4b. For example, the subject of S5, the answers to questions 1a-1c showed a lack of understanding, but in question 4 he was able to solve it as shown in Figure 23. Through the interview, it was found that S5 solved this algebraic question procedurally because he only did a series of procedures without understanding the concept, in this case the meaning of the equals sign and the application of the properties of algebraic operations.

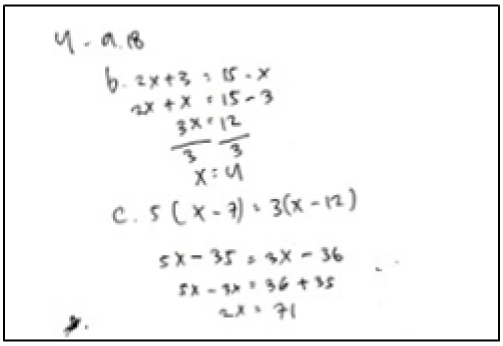


Figure 23 Answers to Algebra 4a-4c by S5

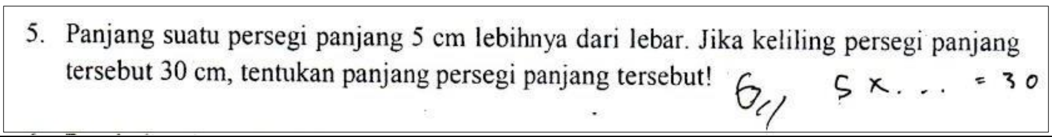
b. Initial algebra questions related to the meaning of the equals sign in algebraic form (questions 2 and 4)

Solve and compare seems to be the dominant strategy chosen by students in solving problem number 2. The subject solves the equation one by one, namely looking for the value of in equation $3 + 7 = 15$ and the value of in equation $3 + 7 - 3 = 15 - 3$, then compare the values. None of the subjects was able to see the relationship between these two equations as equivalent. As for solving problem number 3, only S6 is able to say mathematical sentences that represent the problem as $5 + 4 = + 7$ but verbally (from interviews). Meanwhile, S1 and S3 wrote it in one sentence as 7+2=5+4 (S1) and 5+4=9=7+2=9 (S3) while S4 and S5 wrote it in two different sentences (see table 4.2 indicator column third). All these sentences do not yet indicate there is a problem to be solved. The meaning of the equal sign which means the symbol for writing the answer is still visible on the subject of S2 who wrote it as 2=7+2=9.

c. Problems related to solving problems related to linear equations of one variable (problems 5 and 6)

Almost all subjects had difficulty in solving the fifth and sixth questions in the form of word problems with linear equations of one variable so that not many subjects reached the stage of determining and implementing strategies. However, both of them made a mistake in choosing the formula, namely length times width (the formula for the area of a rectangle) as the formula for the perimeter of a rectangle. In understanding the problem, S3 also made a mistake, namely understanding that 5 cm is the width of a rectangle (Figure 4.24). The S4 has departed from the correct understanding and is able to understand the relationship between two elements, namely the length and width of a rectangle. Unfortunately, there is a wrong process in executing the strategy, namely an error in applying the formula. This error leads to an incorrect final answer.

Likewise for the sixth question, only S3 and S4 were able to independently understand the relationship between the weight of the object on each arm of the scale and were able to solve this problem using a guess and check strategy. Thus, the ability of S3 and S4 to view the problem from a structural perspective has been seen in solving this


5. Panjang suatu persegi panjang 5 cm lebihnya dari lebar. Jika keliling persegi panjang tersebut 30 cm, tentukan panjang persegi panjang tersebut! 6cl 5 x . . . = 30

last problem. As for the other subjects, admitted difficulties in understanding the problems given so that they could not carry out the next process, namely choosing and implementing strategies. Thus, judging from the problem-solving strategy, it does not appear that there are subjects who solve this problem with strategies that involve formal algebra such as modeling equations and solving them to obtain solutions.

Figure 24 Answers to Algebra number 5 by S3

Based on the categorization of problem solving strategies by Hejný, Jirotková & Kratochvilová (2006), the majority of strategies used by students in solving problems about early algebra tend to be included in procedural strategies, because the subject does not involve mathematical structures such as linkages or relationships between elements or application of the nature of the operation.

If it is associated with the results of the analysis of students' relational thinking, it appears that there are some linkages with early algebra problem solving strategies. The strategy used, namely solve and compare, has a tendency to be similar when the subject completes the number sentence by looking at the equality of results. The S3 and S5 subjects who still have a tendency to interpret the equal sign as a symbol to write answers, have not been able to understand the problem given in question 2.

At the completion of the initial algebra test, questions number 5 and 6 which implicitly understand the problem require an understanding of the relationship of numbers. For example, in understanding the statement that the length of the rectangle is 5 cm more than the width, this shows the relationship between numbers. In relation to this indicator, S4 is able to understand the problem in question number 5 (but incorrectly applies the formula) also understands and solves problem number 6. Other subjects who have not been able to use the relationship between numbers in completing number sentences tend to have difficulty or misunderstand (eg. S3 on question 5) questions 5 and 6 on the initial algebra test.

When viewed from the strategy carried out, these three categories are included in the category of procedural strategies (Hejný, Jirotková & Kratochvilová, 2006; Kieran, 2007) because they do not involve an understanding of the mathematical structure. Only the subject of S1 shows relational thinking (structural strategy) about commutative properties and this strategy is also seen in algebra question 1.a, namely by understanding the commutative nature of algebraic forms. Thus, it is suspected that an understanding of the properties of operations on whole numbers seems to help students understand the properties of operations in algebraic forms, because basically arithmetic and algebra come from the same basic idea (Carpenter et al., 2005). This understanding will encourage students to apply structural strategies in solving algebraic problems. This is in line with the results of previous studies that encourage relational thinking is very important because it gives meaning to arithmetic, leads to a conceptual understanding of numbers and the nature of numbers, and improves students' thinking about mathematical generalizations (Jacobs et al, 2007; Molina, Castro, & Mason, 2006; Stephens, 2007).

CONCLUSION

Based on the results of research conducted on six grade VII junior high school students in Bandung, students' relational thinking when completing number sentences only applies to several subjects for each relational thinking indicator, namely the first 3 indicator items: each item is seen in S1, S4, and S6; The second 2 indicator items are visible on the S4; and the third 3 indicator items are seen in S1. In interpreting the equal sign , all subjects were able to interpret it as equality, but in some subjects there was still a tendency to write the equal sign as a symbol to write the next result or answer. In using number relationships and applying the basic nature of number operations, the majority of subjects do computations dominantly and/or apply procedural knowledge-based operations. Similarly, in solving the initial algebraic problem about algebraic forms and PLSV, it was found that the dominant tendency of students to consider algebraic expressions from a procedural perspective rather than a structural perspective (based on mathematical concepts or structures). Thus, if students are given an initial algebra problem on the topic of algebraic forms and PLSV, then students tend to solve the problem from a procedural perspective.

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